BER-Optimal Analog-to-Digital Converters for Communication Links

Minwei Lu, Naresh Shanbhag and Andrew Singer
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, IL 61801
Email: {mlu9, shanbhag, acsinger}@illinois.edu

***Abstract*— In this paper, we propose BER-optimal analog-to-digital converters (ADC) where quantization levels and thresholds are set non-uniformly to minimize the bit-error rate (BER). This is in contrast to present-day ADCs which act as transparent waveform preservers. Simulations for various communication channels show that the BER-optimal ADC achieves shaping gains that range from 2.5dB for channels with low intersymbol interference (ISI) to more than 30dB for channels with high ISI. Moreover, a 3-bit BER-optimal ADC achieves the same or even lower BER than a 4-bit uniform ADC. For flash converters, this corresponds a power reduction by $2\times$. Look-up table based equalizers compatible with BER-optimal ADCs are shown to reduce the power up to 47% and the area up to 66% in a 45nm CMOS process. The shaping gain due to BER-optimal ADCs can be exploited to lower peak transmit swings at the transmitter or decrease power consumption of the ADC.**

I. Introduction

Traditional ADC design is based on a fidelity criterion, attempting to reconstruct the input subject to constraints such as circuit power and process technology. The metric to be optimized, the error between input and output, is captured by signal-to-quantization-noise-ratio (SQNR) and signal-to-noise-plus-distortion-ratio (SNDR). Most ADCs today employ uniform quantization; that is, the levels and thresholds are placed uniformly within the signal dynamic range. As the SQNR depends strongly on the number of bits B_X of the ADC, system design leads one to determine B_X required to meet a specific SQNR or other performance specification. Unfortunately, large values of B_X lead to high power consumption, large area, and increased input capacitance. In high-speed systems, low-power ADCs are particularly difficult to design, and the effective number of bits (ENOB) usually does not exceed 6 [1], [4], [8].

In the context of an ADC-based communication link in Fig. 1(a), we show the eye diagram of the received signal $x_c(nT)$ prior to quantization (Fig. 1(b))along with its probability density function (PDF) (Fig. 1(c)). Signal statistics can be exploited to assign thresholds and levels in the ADC to improve system performance. The problem of determining the optimal set of quantization levels and thresholds was solved in [3] and [5]. The Lloyd-Max algorithm was proposed to iteratively determine the optimal levels $\mathbf{r}$ and thresholds $\mathbf{t}$ of a quantizer. We show in this paper that the Lloyd-Max algorithm improves SQNR in communication links but does not necessarily reduce BER.

Hence, we propose an ADC in which the levels and the thresholds are set to minimize the BER. We term this a BER-optimal or BER-aware ADC because it employs a detection criterion and, instead of SQNR, maximizes the probability of detecting a transmitted bit correctly. The idea of BER-optimal components is not novel, as BER-optimal equalizers [2], [10] and sampling phase [2] have been determined. However, this is the first work which addresses the issue of designing BER-optimal ADCs. BER-optimal ADCs differ from various digitally-assisted ADCs [6], [7] as the latter maximize SQNR.

The rest of this paper is organized as follows. Section II presents an algorithm for computing BER-optimal levels and thresholds. Section III compares the performance of the BER-optimal and traditional ADCs via simulations for different channels.

II. BER-Optimal ADC

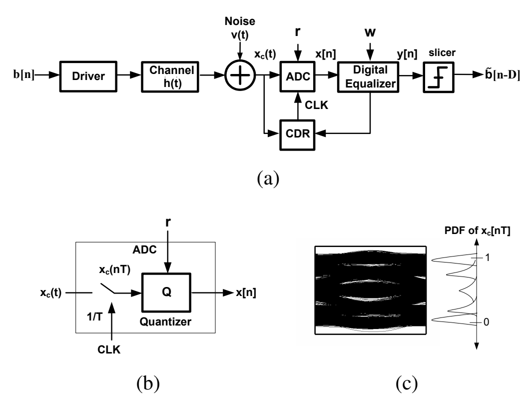


Fig. 1. Role of an ADC in a communication link: a) block diagram of a communication link, b) functional diagram of an ADC, and c) eye diagram and PDF of the sampled received signal $x_c(nT)$.

Fig. 1(a) illustrated a linearly equalized communication link. Assuming binary phase shift keying (BPSK) modulation, the transmitter sends pseudorandom sequence of bits $b[n] \in \{\pm 1\}$ through the channel. At the receiver, the ADC quantizes the signal, and the outputs are subsequently processed by a digital equalizer to eliminate ISI that results from the channel. A slicer following the equalizer makes a hard decision on which bit has been transmitted. With a slight abuse of notation, we

978-1-4244-5309-2/10/$26.00

refer to the BPSK symbols as bits in the sequel. As shown in Fig. 1(b), the ADC consists of a baud-rate sampler followed by a quantizer. In this paper, we focus on determining BER-optimal quantizer parameters.

At a given sampling time index n, the input $x_c[n] = x_c(nT)$ to the ADC is given by

$$x_c[n] = \sum_{i=0}^{M-1} h[i]b[n-i] + v[n] \quad (1)$$

where $b[n]$ is the transmitted bit, $h[i]$ the baud-rate sampled impulse response of the channel with memory M, and $v[n]$ is modeled as additive white Gaussian noise with variance σ^2.

The ADC has N levels r_k ($k = 1, \ldots, N$) and $N-1$ thresholds t_k ($k = 1, \ldots, N-1$), where N is equal to 2^{B_x}. The mapping between $x_c[n]$ and the quantized signal $x[n]$ is

$$\begin{aligned} x[n] &= r_1 \text{ if } x_c[n]\epsilon(-\infty, t_1] \\ &= r_N \text{ if } x_c[n]\epsilon(t_{N-1}, \infty) \\ &= r_k \text{ if } x_c[n]\epsilon(t_{k-1}, t_k] \text{ for } k = 2, \ldots, N-2. \end{aligned} \quad (2)$$

The output of the L-tap linear equalizer (LE) will be the convolution of ADC outputs and equalizer coefficients $\mathbf{w}$. The estimate of the transmitted symbol $b[n-D]$ is $\tilde{b}[n-D] = \text{sgn}(y[n])$. Here D accounts for delay in the channel and equalizer; it must be chosen carefully to achieve good BER.

A. Uniform ADC

In a uniform ADC, the quantization levels are spread evenly within the signal dynamic range. The minimum and maximum input amplitudes expected by this ADC are expressed as $-V_{max}$ and V_{max}, respectively. The quantizer step-size is $\Delta = \frac{2V_{max}}{N} = \frac{2V_{max}}{2^{B_x}}$. For sufficiently small quantization error, $q[n] = x_c[n] - x[n]$ is assumed to be a uniformly distributed random variable, bounded between $-\frac{\Delta}{2}$ and $+\frac{\Delta}{2}$ and independent of input. Quantization noise power σ_q^2 is given by $E[q^2[n]] = \frac{\Delta^2}{12}$. For uniform quantization, SQNR can be calculated from $6.02B_X + 4.8 - 20\log_{10}\frac{V_{max}}{\sigma_x}$, where each additional bit increases SQNR by 6dB.

B. Non-uniform ADC Lloyd-Max Quantizer

A Lloyd-Max Quantizer [3], [5] minimizes the distortion measure known as the mean-squared error $E[q^2[n]]$ (MSE), given by

$$\begin{aligned} E(q^2) &= E[(x_c - r_k)^2] \\ &= \sum_{k=1}^{N} \int_{t_{k-1}}^{t_k} (x_c - r_k)^2 f_{X_c}(x_c) dx_c \end{aligned} \quad (3)$$

where X_c is the random variable representing input $x_c[n]$, and $f_{X_c}(x_c)$ is its PDF.

Stationary points of the MSE in terms of $\mathbf{r}$ and $\mathbf{t}$ can be found by differentiation with respect to $\mathbf{r}$ and $\mathbf{t}$:

$$r_{k,opt} = \frac{\int_{t_{k-1,opt}}^{t_{k,opt}} x_c f_{X_c}(x_c) dx_c}{\int_{t_{k-1,opt}}^{t_{k,opt}} f_{X_c}(x_c) dx_c}, \quad (4)$$

$$t_{k,opt} = \frac{r_{k,opt} + r_{k+1,opt}}{2}. \quad (5)$$

These equations are often difficult to solve, so the Lloyd-Max algorithm iteratively determines $\mathbf{r}$ and $\mathbf{t}$. Although this algorithm improves SQNR, it is not the same as minimizing BER.

C. BER-Optimal ADC

We propose quantization based on the detection criterion, by setting the levels $\mathbf{r}$ and thresholds $\mathbf{t}$ non-uniformly using the BER metric. In the system presented in Fig. 1(a), an error is made when $\tilde{b}[n] \neq b[n]$ (assuming $D = 0$), so BER is computed by averaging over all possible values of $y[n]$ and hence all vectors $\mathbf{x}_n = [x[n], x[n-1], ..., x[n-L+1]]$ such that $\tilde{b}[n] = \text{sgn}(y[n]) = \text{sgn}(\mathbf{w}^T\mathbf{x}_n)$ produces an error at the slicer,

$$\begin{aligned} BER &= \mathbb{P}\{b[n] \neq \tilde{b}[n]\} \\ &= \sum_{y[n]} \left[\mathbb{P}\{y[n]\} \left(\frac{1 - b[n]\tilde{b}[n]}{2} \right) \right] \\ &= \sum_{\mathbf{x}_n} \left[\left(\prod_{j=0}^{L-1} \mathbb{P}\{x[n-j] = r_k\} \right) \left(\frac{1 - b[n]\tilde{b}[n]}{2} \right) \right] \end{aligned} \quad (6)$$

where $\mathbb{P}\{x[n-j] = r_k | x_{c0}[n-j]\}$ is given by

$$Q\left(\frac{t_{k-1} - x_{c0}[n-j]}{\sigma}\right) - Q\left(\frac{t_k - x_{c0}[n-j]}{\sigma}\right), \quad (7)$$

$\mathbb{P}\{\bullet\}$ signifies the probability of an event, $Q(\bullet)$ is the Gaussian Q function, and $x_{c0}[n]$ is noiseless channel output $\sum_{i=0}^{M-1} h[i]b[n-i]$. A BER-optimal ADC is one where $\mathbf{r}$ and $\mathbf{t}$ minimize (6).

A closed form expression for the BER optimal parameters of the ADC, $\mathbf{r}$ and $\mathbf{t}$, is difficult to obtain due to the highly non-linear objective function. Therefore, we employ the gradient descent algorithm to determine the parameters. The following update equations are used to compute $\mathbf{r}$ iteratively. For the ith iteration of the algorithm, we have

$$\begin{aligned} BER &= f(\mathbf{h}, \mathbf{r}, \mathbf{t}, \mathbf{w}, \sigma) \\ \mathbf{r_i} &= \mathbf{r_{i-1}} + \mu \left(\frac{\partial BER}{\partial \mathbf{r}} \right) |_{\mathbf{r} = \mathbf{r_{i-1}}} \\ &\approx \mathbf{r_{i-1}} + \mu \left(\frac{\Delta BER}{\Delta \mathbf{r}} \right) \end{aligned} \quad (8)$$

The placement of $\mathbf{t}$ remains the same as given by (5). To avoid differentiating the sign function, the gradient is computed by finite differences–each entry in the gradient vector is obtained by perturbing the r_k's one at a time and computing the change in BER due to this perturbation [9].

This algorithm can readily be extended to decision-feedback equalizers. Simply replace $\tilde{b}[n] = \text{sgn}\left(\sum_{j=0}^{L-1} w[j]x[n-j]\right)$ with $\text{sgn}\left(\sum_{j=0}^{L-1} w[j]x[n-j] - \sum_{l=1}^{L_2} d[l]\tilde{b}[n-l]\right)$, where $\mathbf{w}$ is the vector containing coefficents of the feedforward filter, and $\mathbf{d}$ contains L_2 coefficents of the feedback filter.

We demonstrate next in simulations that the BER-optimal ADC outperforms the uniform and Lloyd-Max quantization approaches.

III. Simulation Results

This section presents simulation results for several backplane-like channels with different levels of ISI.

A. Simulation Methodology

First, given a channel impulse response, a minimum mean squared error (MMSE) linear equalizer with three taps is obtained assuming a uniform ADC. Next, (8) was used to iteratively approximate the minimum BER thresholds and representation levels for the ADC. Equation (6) was then used to compute the BER analytically. We verified our expressions via Monte-Carlo simulations and error counting for BER down to 10^{-7}. In order to isolate the effect of nonuniform quantization, the equalizers in all setups are MMSE linear equalizers with 3 taps. In addition, only equalizer inputs are quantized; the equalizer itself has infinite precision. Signal-to-noise ratio (SNR) was computed by SNR $= \sum_{i=0}^{M-1} \frac{h[i]^2}{\sigma^2}$.

We define the *ADC shaping gain* as $S_G(BER) = SNR_{old}(BER) - SNR_{new}(BER)$ to quantify the reduction in SNR achieved via the BER-optimal techniques.

B. BER-Optimal ADC vs. Uniform ADC

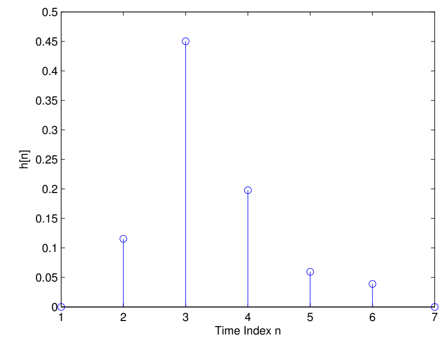
This subsection presents results for two backplane-like channels with different levels of ISI, where ISI is quantified by the ratio between the squared magnitude of the main tap to the sum of the squared magnitudes of the smaller taps of the impulse response.

1) Channel with low level of ISI (Fig. 2(a)): Fig. 2(b) shows that a 3-bit BER-optimal ADC performs better than a 3-bit uniform ADC. Furthermore, a 3-bit BER-optimal ADC is as effective or better than a 4-bit uniform ADC. The BER curve for an infinite precision ADC, infinite precision equalizer is also displayed for comparison purposes. In both the low and high SNR regimes (BER=10^{-4} and 10^{-15}, respectively), the shaping gain S_G achieved by the BER-optimal ADC is $2.5dB$.

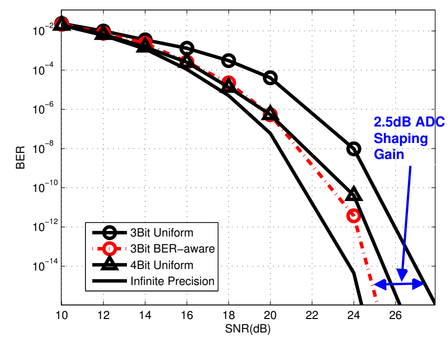
2) Channel with high level of ISI (Fig. 3(a)): When channels with high levels of ISI are employed for testing, the 3-bit BER-optimal ADC is significantly better than the 3-bit uniform ADC as shown in Fig. 3(b). In this case, performance of the 3-bit uniform ADC does not improve with increasing SNR due to severe quantization noise. Compared to a 3-bit uniform ADC, ADC shaping gain S_G is too large to be quantified; compared to a 4-bit uniform ADC, $S_G(BER = 10^{-15}) = 3$dB .

C. BER-Optimal ADC vs. Lloyd-Max ADC

Although a Lloyd-Max ADC can improve SQNR, Fig. 4 shows that a 2-bit Lloyd-Max ADC followed by a MMSE linear equalizer results in little BER improvement when compared with a 2-bit uniform ADC followed by a MMSE LE. This observation indicates that SQNR is not the best metric when the goal is to reduce BER. In contrast, a receiver based on the detection criterion (2-bit BER-optimal ADC followed by min-BER linear equalizer, where the equalizer coefficients are computed in a similar manner as in (8) using gradient descent algorithm), results in significant improvement, surpassing even a 3-bit uniform ADC for SNR $> 16dB$. This demonstrates that the detection criterion is a more effective metric than the fidelity criterion in communication links.



(a)



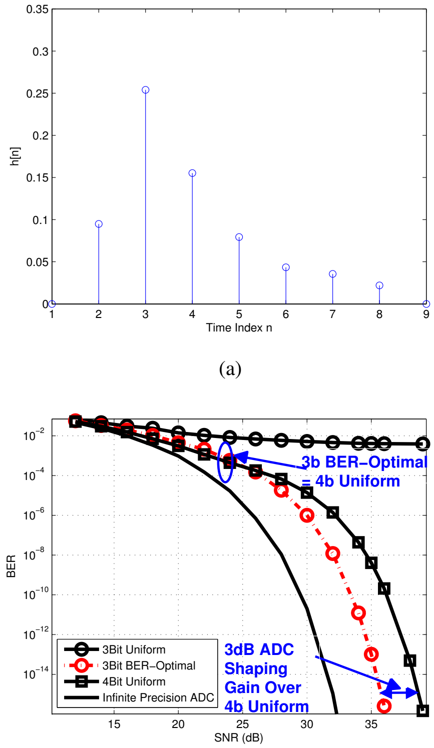
(b)

Fig. 2. Performance for a low-ISI channel: a) sampled impulse response of a backplane-like channel, and b) BER vs. SNR curves for a 3-bit uniform, 3-bit BER-optimal, 4-bit uniform, and infinite-precision ADC, respectively.

D. Feasibility Study

Since the quantization levels are no longer equidistant, a change in digital output will correspond to different changes in analog input. The equalizer cannot operate directly on such digital outputs. Thus, to conduct a first order feasibility study of the BER-optimal ADCs, we synthesized the digital equalizer following the BER-optimal ADC and compared its complexity to that of the standard linear equalizer.

The equalizer architecture is obtained by employing a look-up table (LUT). This is done by mapping the ADC bits in the tapped-delay line directly to a binary value corresponding to the detected bits. The channels are those presented in Section III-B; for each channel, two design points in the plots, corresponding to low and high input SNRs, are synthesized and compared. The BER-optimal ADCs have 3 bits, while the benchmark is a 4-bit uniform ADC, 3-tap linear equalizer, with



(a)

(b)

Fig. 3. Performance for a high-ISI channel: a) sampled impulse response of a backplane-like channel, and b) BER vs. SNR curves for a 3-bit uniform, 3-bit BER-optimal, 4-bit uniform, and an infinite-precision ADC, respectively.

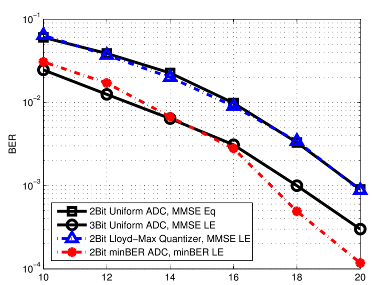


Fig. 4. Performance comparison between the BER-optimal and Lloyd-Max ADC for a synthetic channel $h = [0.1\ 0.7\ 0.4]$.

sufficient bits assigned to equalizer coefficients to avoid BER degradation due to coefficient quantization at BER of 10^{-4}. From Table I, we see that the LUT-based equalizer is in fact much simpler than the conventional FIR filter, indicating BER-optimal ADCs are superior. The area and power numbers are provided by synthesis reports from Nangate's Open 45nm Cell Library. At low SNR design point for the high ISI channel, the standard LE occupies $269.7\mu m^2$, while area of the LUT-based equalizer is only $91.5\mu m^2$. This is a reduction of 66%. To summarize, for low SNRs, area of the LUT-based equalizer reduces by 55% to 66%, and power reduces by about 45%. For high SNRs, area of the LUT-based equalizer reduces by 39% to 56%, and power reduction is around 24% to 32%. Global voltage of 0.95V and clock frequency of $400MHz$ are used.

TABLE I
COMPARING COMPLEXITY OF LUT-BASED EQUALIZER WITH LE.

Low ISI Channel	**Fig. 2(a)**	
SNR (dB)	**Cell Area** (μm^2)	**Power** (μW)
10 (LUT)	108	22.2
10 (LE)	244.4	40.5
18 (LUT)	106	22.7
18 (LE)	177	29.9
High ISI Channel	**Fig. 3(a)**	
SNR (dB)	**Cell Area** (μm^2)	**Power** (μW)
12 (LUT)	91.5	22.6
12 (LE)	269.7	43.0
24 (LUT)	93	23.1
24 (LE)	209	34.4

IV. CONCLUSION

This paper presents a BER-optimal ADC that requires less precision, hence lower power, that conventional ADCs while achieving the same BER. Future work can focus on studying the benefits of this approach over a variety of channels, modulation schemes, and receiver architectures.

V. ACKNOWLEDGMENT

The authors acknowledge the support of the Gigascale System Research Center (GSRC), one of five research centers funded under the Focus Center Research Program (FCRP), a Semiconductor Research Corporation program and Texas Instruments, Inc.

REFERENCES

[1] H.-M. Bae, J. B. Ashbrook, J. Park, N. R. Shanbhag, A. C. Singer, and S. Chopra. "MLSE receiver for electronic dispersion compensation of OC-192 fiber links". *IEEE Journal of Solid State Circuits*, 41(11):2541–2554, 2006.

[2] E.H. Chen et al. "Near-Optimal Equalizer and Timing Adaptation for I/O Links Using a BER-based Metric". *IEEE Journal of Solid-State Circuits*, 43(9):2144–2156, 2008.

[3] S. Lloyd. "Least Squares Quantization in PCM". *IEEE Transactions on Information Theory*, IT-28(2):129–137, 1982.

[4] M. Harwood et al. "A 12.5 Gb/s SerDes in 65nm CMOS using a baud-rate ADC with digital RX equalization and clock recovery". In *IEEE International Solid-State Circuits Conference*, 2007.

[5] J. Max. "Quantizing for Minimum Distortion". *IEEE Transactions on Information Theory*, IT-6:7–12, 1960.

[6] A. Meruva and B. Jalali-Farahani. "A 14-b 32MS/s Pipelined ADC with Novel Fast-Convergence Comprehensive Background Calibration". In *IEEE International Symposium on Circuits and Systems*, pages 956–959, 2009.

[7] P. Nikaeen and B. Murmann. "Digital Compensation of Dynamic Acquisition Errors at the Front-End of High-Performance A/D Converters". *IEEE Journal of Selected Topics in Signal Processing*, 3(3):499–508, 2009.

[8] P. Schvan et al. "A 24GS/s 6b ADC in 90nm CMOS". In *IEEE International Solid-State Circuits Conference*, 2008.

[9] William H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. *Numerical Recipes: The Art of Scientific Computing*. Cambridge University Press, NY, 2007.

[10] C.-C. Yeh and J. R. Barry. "Adaptive minimum bit-error rate equalization for binary signaling". *IEEE Transactions on Communication*, 48(7):1226–1235, 2000.