Reduced Precision Redundancy for Low-power Digital Filtering *

Byonghyo Shim and Naresh R. Shanbhag
Coordinated Science Laboratory, ECE Dept.
University of Illinois at Urbana-Champaign
1308 West Main Street, Urbana, IL 61801
Email: [bshim,shanbhag]@mail.icims.csl.uiuc.edu

Abstract

In this paper, we propose a low-power digital filtering technique based on voltage overscaling (VOS) and a novel algorithmic noise-tolerant (ANT) technique referred to as reduced precision redundancy (RPR). VOS implies scaling of the supply voltage beyond the critical voltage required for correct operation. RPR involves having a reduced precision replica whose output can be employed as the final output in case the original filter computes erroneously. In addition, an LSB estimator is also employed to compensate for information loss in the LSBs. For frequency selective filtering, it is shown that the proposed technique provides a 30% energy savings over an optimally scaled (i.e., the supply voltage equals the critical voltage) present day system. Energy savings of up to 65% can be achieved if a SNR loss of 1.5 dB is tolerated

1 Introduction

Rapid growth of wireless communication industry in recent years has made power dissipation a great issue for DSP system designs. Since the power dissipation is proportional to the square of the supply voltage, supply voltage scaling techniques have been widely used to obtain the substantial energy reduction [1,2]. However, since the delay of circuit elements becomes slower as supply voltage is scaled down, the extent of voltage scaling is limited by a minimum supply voltage $V_{dd-crit}$ at which the throughput requirement is just satisfied.

Recently, voltage overscaling (VOS) has been proposed as a means of signficant energy savings [3,4]. Voltage overscaling refers to reduction of supply voltage beyond $V_{dd-crit}$, without sacrificing the throughput, to

$$V_{dd} = K_{vos} V_{dd-crit}, \ 0 \leq K_{vos} < 1 \qquad (1)$$

where K_{vos} is voltage-overscale factor. However, since the critical path delay T_{cp} of the system becomes greater than the clock period T_s, input-dependent intermittent soft errors occur whenever paths longer than T_s are excited. To compensate for these errors, algorithmic noise-tolerance (ANT) techniques have been proposed as an efficient error control scheme [3, 4]. Besides VOS, since the soft errors generated by the increased clock speed have similar characteristics, ANT schemes can be applied to high throughput applications with marginal penalty in algorthmic performance. Further, this technique can be used to mitigate the effects of deep submicron noise and process variations [3].

In this paper, we propose a novel ANT technique based on reduced-precision redundancy (RPR) which is efficient in error correction when voltage-overscaling is applied. By using the RPR filter which generates MSB estimate of original filter, we correct errors generated by the critical path excitation. The concept of the RPR is described in the next section. The digital FIR filtering and design of LSB estimator to compensate for the information loss is presented in section 3. In section 4, the simulation results are presented and conclusions are presented in section 5.

2 Error Control through Reduced Precision Redundancy

In this section, we present an algorithmic noise-tolerance scheme for DSP systems that compensates for errors caused by voltage overscaling. The proposed soft DSP system is shown in Fig. 1, where a reduced precision system is added to the voltage overscaled noisy system to detect and correct errors. We first describe the error characteristics of the noisy system and then present proposed the scheme.

2.1 Error Characteristics

Voltage overscaling introduces input-dependent soft errors whenever a path with delay greater than clock period T_s is excited. The soft errors appear first in the most significant bit (MSB) as the arithmetic units employed in DSP systems use least significant bit (LSB) first computation. This creates errors having large magnitude. Though large errors degrade performance severely, they can be easily detected if we know the information about correct MSBs for error

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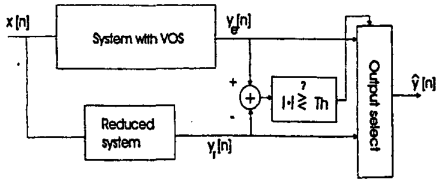

Figure 1: Overview of proposed algorithmic noise tolerance (ANT) scheme using reduced precision redundancy

detection.

2.2 Reduced Precision Redundancy

The RPR system is a replica of the original system with reduced precision operands. The output of RPR has small errors due to loss of LSB information in the operands. However, since the soft errors due to VOS occur mainly in MSBs, we can use the output of the RPR to detect errors in the output of the original noisy system. If we detect an error, as shown in Fig. 1, the output of the RPR is declared as an actual output.

It is assumed above that the RPR does not suffer from soft errors. This assumption is valid as the critical path delays of adders and multipliers decrease linearly with decrease in precision. For example, if the precision is halved in a 0.25 μm CMOS technology, supply voltage can be reduced upto $K_{vos} = 0.44$ before the critical path of the reduced precision system is violated. Further, to achieve the goal of reducing power while maintaining the system performance, for a given K_{vos}, the smallest precision that achieves the required performance should be chosen. This minimizes the area overhead and additional power consumption of the reduced precision system.

Precision reduction in full system has been proposed as a low power technique previously [5], but this technique has disadvantage of allowing the performance loss corresponding to the precision reduction. However, by employing RPR as an error controller for the system with VOS, proposed technique achieves substantial power reduction with just negligible performance loss.

2.3 Threshold Selection & Decision rule

Let $y_o[n]$ denote the output of original noisy system and $y_r[n]$ be the output of reduced precision system. In the case of FIR filter, the difference between noisy filter output and reduced filter output is obtained:

$$y_o[n] - y_r[n] = \sum_{k=0}^{N-1} h_o[k]x_o[n-k] - \sum_{k=0}^{N-1} h_r[k]x_r[n-k] \tag{2}$$

Defining the filter coefficient and input error terms $h_e = h_o - h_r$ and $x_e = x_o - x_r$ respectively, we get

$$y_o[n] - y_r[n] = \sum_{k=0}^{N-1} h_e[k]x_o[n-k] + \sum_{k=0}^{N-1} h_r[k]x_e[n-k] \tag{3}$$

In case of no errors in operations, the quantity in eq. (3) is the quantization noise of the reduced precision. Hence, we can use this difference as a reference for the error detection. It can be shown that the absolute difference is bounded as:

$$|y_o[n] - y_r[n]| \leq T \tag{4}$$

where T is a threshold given by

$$T = \sum_{k=0}^{N-1} |h_e[k]|\cdot|x_{o,max}[n-k]| + \sum_{k=0}^{N-1} |h_r[k]|\cdot|x_{e,max}[n-k]| \tag{5}$$

Since this bound is obtained by using the maximum of inputs, a small and easy implementable threshold can be chosen in real applications.

If the distance between two system outputs is less than the threshold T, we can decide that no soft-error or small error has occurred. Instead, if distance is larger than the T, we determine that the output has a soft error. In case of an error detection, we use the output of the reduced precision system as the corrected one, otherwise, we use the output of original system. The decision rule for choosing the system output is then given by

$$\hat{y}_n = \begin{cases} y_o[n] & \text{if } d(y_e, y_r) \leq T \\ y_r[n] & \text{if } d(y_e, y_r) > T \end{cases} \tag{6}$$

where $d(y_e, y_r) = |y_e - y_r|$.

3 FIR Filtering of RPR System

The FIR filter structure employing the proposed RPR scheme is shown in Fig. 2. Both the original system and the RPR use multiply-and-accumulate (MAC) structures. Note that the LSB estimator is incorporated in the RPR to compensate for information loss of the LSB part. For the purpose of simulation, we considered a lowpass FIR filter given in eq. (7) for frequency selective filtering.

$$h[n] = \frac{sin[\omega_c n]}{\pi n} \cdot \omega[n] \tag{7}$$

where $\omega[n]$ is a symmetric N-point Hamming window function.

3.1 Optimal Threshold Search

As mentioned previously, since the bound we get from eq. (5) is not such tight, it may not produce best performance in real applications. The performance of

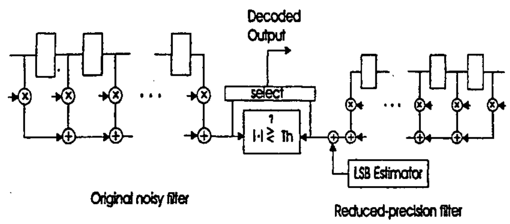

Figure 2: Proposed ANT scheme using reduced precision redundancy

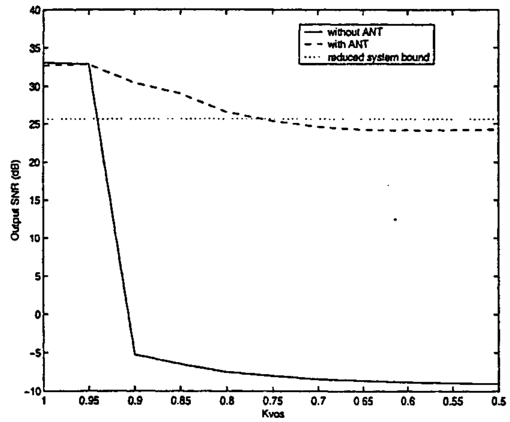

Figure 3: Performance of proposed algorithmic noise tolerance (ANT) system employing reduced precision redundancy

the FIR lowpass filter with the proposed ANT scheme and noisy system without ANT is compared in Fig. 3. As shown in the figure, ANT scheme shows far better performance than the noisy system without ANT. But the performance degrades as K_{vos} increases due to incorrect error decision. Thus, instead of using this bound as the error-detection threshold, it is desired to find a more accurate threshold. Figure 4 shows the performance of proposed ANT system for the variations of error detection threshold. If threshold is set too large the system output shows a tendency to follow the noisy system output. Contrary, if it is set too small, most of the system output will be that of reduced precision system. Between these extremes, an optimal threshold that achieves the best performance for a given filter can be found.

3.2 LSB estimator

The RPR based error detection scheme has the disadvantage of not conserving the least significant bit (LSB) information of original system. Though there is little additional power consumption and area overhead in the RPR block, inexact outputs due to the loss of LSB information may result in SNR degradation. As

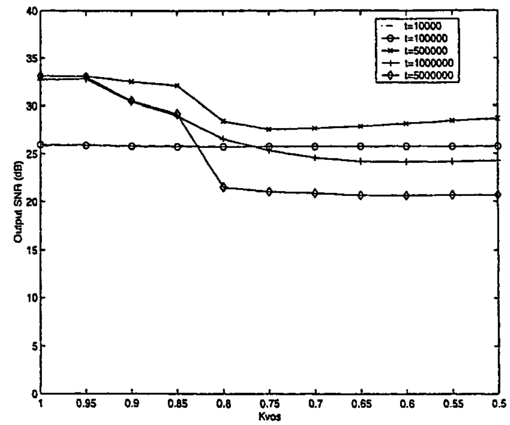

Figure 4: Performance of ANT system for the threshold variation

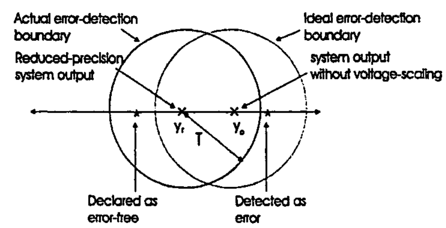

Figure 5: Error-detection boundary of reduced system

depicted in Fig. 5, the error detection boundary is centered around the output of the reduced precision system. Due to this bias, outputs with smaller distance from the correct system output may be detected as an error. Similarly, outputs with larger distance from original one may be declared as an error-free. To alleviate the incorrect decision and therefore enhance the error detection capability, we employed LSB estimation term which compensates for the LSB information of the reduced precision systems. The modified output of RPR with LSB estimator is given by

$$y_r' = y_r + \Delta \quad (8)$$

where Δ is the LSB correction term. Suppose the system performs K-tap FIR filtering and the original system and reduced system have n-bit and l-bit wordlengths respectively. In two's complement multiplications, the original multiplication is expressed as

$$x_k \cdot h_k = (-x_{k,n-1}2^{n-1} + \sum_{m=0}^{n-2} x_{k,m}2^m) \cdot (-h_{k,n-1}2^{n-1} + \sum_{m=0}^{n-2} h_{k,m}2^m) \quad (9)$$

On the other hand, the multiplication of reduced system can be expressed as

$$x_k^r \cdot h_k^r = (-x_{k,n-1}2^{n-1} + \sum_{m=n-l}^{n-2} x_{k,m}2^m) \cdot (-h_{k,n-1}2^{n-1} + \sum_{m=n-l}^{n-2} h_{k,m}2^m) \quad (10)$$

The computation error between two system is given by

$$\varepsilon \simeq \sum_k [(-x_{n-1}2^{k,n-1} + \sum_{m=n-l}^{n-2} x_{k,m}2^m) \sum_{m=0}^{n-l-1} h_{k,m}2^m + \sum_{m=0}^{n-l-1} x_{k,m}2^m(-h_{k,n-1}2^{n-1} + \sum_{m=n-l}^{n-2} h_{k,m}2^m)] \quad (11)$$

The objective in LSB estimator design is to minimize information loss in the sense that overall distortion between lost information and LSB estimation term is minimal [6]. There the distortion is given by

$$d^2(\varepsilon, \Delta_j) = \int_{S_j} \sum_{i=1}^{K} (\varepsilon_i - \Delta_j)^2 f_\varepsilon(\varepsilon) d\varepsilon \; j = 1, ..., L. \quad (12)$$

where Δ_i is the i-th LSB estimation term, $f_\varepsilon(\varepsilon)$ is probability density function of error, and S_j is the supporting region of j-th estimation term.

The procedure of obtaining LSB estimation term which generates minimum distortion when it goes to its limit is summarized as follows,

- Step I : Determine the LSB estimation level L and choose ordered set $\{\Delta_j\}$.
- Step II : Set the following non-overlapping regions halfway between neighbor estimator terms.

$$S_k = [\frac{\Delta_{k-1} + \Delta_k}{2}, \frac{\Delta_k + \Delta_{k+1}}{2}) : K = 1, .., L \quad (13)$$

- Step III : Find expectation for each non-overlapping region,

$$\Delta_k = E[\varepsilon | \varepsilon \in S_k] = \frac{\int_{S_k} \varepsilon \cdot f_\varepsilon(\varepsilon) d\varepsilon}{\int_{S_k} f_\varepsilon(\varepsilon) d\varepsilon} \quad (14)$$

- Step IV : Repeat from step II until distortion term converges.

Figure 6 shows the simulation results for the reduced precision system incorporating LSB estimator. We can observe that the output SNR increases as the level

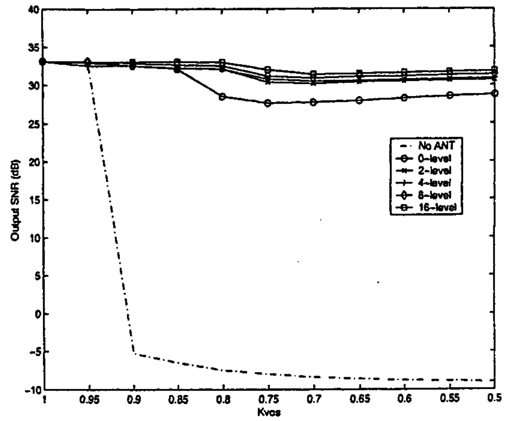

Figure 6: Performance of the LSB estimator

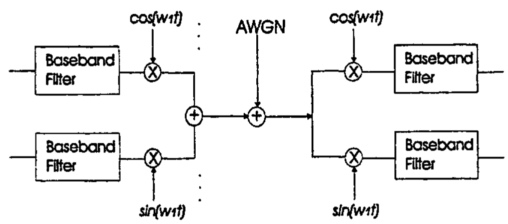

Figure 7: Receiver baseband filtering by proposed RPR based ANT scheme

of LSB estimator increases. However, since the performance saturates as the number of estimation level becomes more than four, it is sufficient to use a 4-level estimator. Hence, with just negligible overhead of logic gates or lookup table for storing LSB estimation term, marginal improvement in system performance can be achieved.

4 Simulation Results

The output SNR, SNR_{ANT}, of the overall system employing RPR filter is defined as

$$SNR_{ANT} = 10log(\frac{\sigma_s^2}{\sigma_n^2 + \sigma_{vos}^2}) \quad (15)$$

where σ_s^2, σ_n^2, and σ_{vos}^2 are the power of desired signal, wideband noise and the soft error introduced by VOS respectively. As shown in Fig. 7, the situation is to extract the primary baseband signal $s_i[n]$ from adjacent bandpass signal $s_j[n], j \neq i$ and wideband Gaussian noise $w[n]$. We assume all the signals and noise are statistically independent from each other. The MAC structure of proposed ANT system is shown in Fig. 8. Note that the MAC of RPR including an LSB estimator is added to the original system. The overhead of the proposed scheme includes the reduced precision

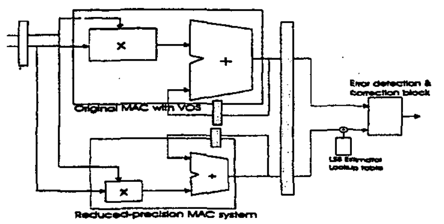

Figure 8: Multiply-and-accumulate (MAC) architecture of proposed ANT system

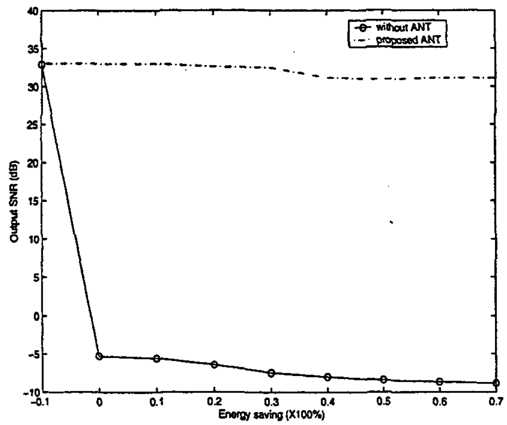

Figure 9: Energy savings vs. SNR plot of the proposed ANT scheme in 0.25 μm CMOS technology with $\alpha = 1.2$

MAC with small-size lookup table and a comparator for error detection. Since the soft errors are typically of large magnitude, we can use a small precision subtractor and multiplexer as a comparator. We assume that the precision of the received signal is quantized by an A/D converter to have a fixed precision wordlength of 13 bits and the wordlength of RPR is set to 8 bits. To prevent the internal overflow of MAC operation and retain the dynamic range in intermediate steps of computations, we used 3 guard bits in the accumulator. To estimate the energy savings obtained via voltage scaling, we used the gate-level power simulator MED [7]. The energy savings E_{sav} is given by

$$E_{sav} = \frac{E_{org} - E_{ANT}}{E_{org}} \times 100\% \qquad (16)$$

where E_{org} and E_{ANT} are the energy dissipations of conventional method and the proposed ANT scheme respectively.

Figure 9 shows the energy vs. performance relationship of the proposed ANT scheme employing reduced precision redundancy. While the conventional system has sharp SNR drop as supply voltage is reduced, the proposed ANT scheme maintains the performance until about 30% reduction in energy without negligible performance loss (0.3 dB). Further, if about 1.5 dB performance is tolerated, then 65% energy saving can be achieved maximally. Thus, we can observe a trade-off between the algorithmic performance and achievable energy savings.

5 Conclusions

In this paper, we have proposed a new ANT technique based on RPR for low-power digital filtering. By employing the RPR which estimates the MSB part of original system, we could correct the soft error caused by VOS. In addition, to enhance the performance by compensating for the LSB information, we incorporated the LSB estimator. It was shown that substantial energy savings can be obtained in frequency selective digital filtering through the proposed RPR technique. Future work includes hardware overhead reduction and improvement in algorithmic performance. Application of RPR to other widely used DSP algorithms and systems are also being investigated.

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